### The Plurality and Borda Count Methods Lecture 9 Sections 1.1 - 1.3

Robb T. Koether

Hampden-Sydney College

Fri, Feb 2, 2018

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### The Math Club Election

- 3 The Plurality Method
- The Borda Count Method
- 5 Burying a Candidate
- 6 Assignment

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# Outline

### Definitions

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### **Definition (The Candidates)**

The candidates are the people running for office in an election. If we are choosing something other than people, we call them alternatives.

### **Definition (The Voters)**

The voters are the people who have a say in the outcome of the election. All votes count equally.

### **Definition (Single-choice Ballot)**

In a single-choice ballot, each voter selects one candidate.

#### **Definition (Preference Ballot)**

In a preference ballot, each voter ranks all the candidates from most preferred to least preferred.

#### Definition (Truncated Preference Ballot)

In a truncated preference ballot, each voter ranks some, but not all, the candidates by preference.

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#### **Definition (Preference Ballot)**

In a preference ballot, each voter ranks all the candidates from most preferred to least preferred.

#### Definition (Truncated Preference Ballot)

In a truncated preference ballot, each voter ranks some, but not all, the candidates by preference.

• We will use preference ballots (also called ranked choice ballots).

# Outline

### Definitions

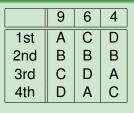


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- There are four candidates for Math Club president: A, B, C, and D.
- There are 19 voting members. Their preferences are shown on the next slide.

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The preferences.

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- Who should be elected president?
- Who is more popular, A or B?
- Who is more popular, A or C?
- Who is more popular, A or D?
- Who is least popular?

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- Who should be elected president?
- Who is more popular, A or B?
- Who is more popular, A or C?
- Who is more popular, A or D?
- Who is least popular?
- Do "least popular" and "most unpopular" mean the same thing?

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### **Definition (The Plurality Method)**

By the plurality method, the candidate with the most *first-place* votes wins.

#### Example

In the Debate Club example, A wins by the plurality method.

Web Page

Run the program Voting Methods on the web.

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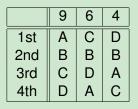
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### **Definition (The Borda Count Method)**

By the Borda count method, the voters rank the candidates. Then each rank is assigned points, higher ranks receiving more points. The candidate with the *most points* wins.

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 Reconsider the Math Club election with 4 points for 1st, 3 for 2nd, 2 for 3rd, and 1 for 4th.



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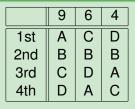
	9	6	4
1st	Α	С	D
2nd	В	В	В
3rd	С	D	А
4th	D	А	С

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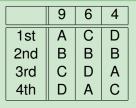


#### Points for $A: 9 \times 4 + 6 \times 1 + 4 \times 2 = 36 + 6 + 8 = 50$ .

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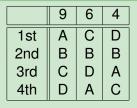


Points for  $A: 9 \times 4 + 6 \times 1 + 4 \times 2 = 36 + 6 + 8 = 50$ . Points for  $B: 9 \times 3 + 6 \times 3 + 4 \times 3 = 27 + 18 + 12 = 57$ .

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Points for  $A: 9 \times 4 + 6 \times 1 + 4 \times 2 = 36 + 6 + 8 = 50$ . Points for  $B: 9 \times 3 + 6 \times 3 + 4 \times 3 = 27 + 18 + 12 = 57$ . Points for  $C: 9 \times 2 + 6 \times 4 + 4 \times 1 = 18 + 24 + 4 = 46$ .

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	9	6	4
1st	Α	С	D
2nd	В	В	В
3rd	С	D	Α
4th	D	А	С

Points for  $A : 9 \times 4 + 6 \times 1 + 4 \times 2 = 36 + 6 + 8 = 50$ . Points for  $B : 9 \times 3 + 6 \times 3 + 4 \times 3 = 27 + 18 + 12 = 57$ . Points for  $C : 9 \times 2 + 6 \times 4 + 4 \times 1 = 18 + 24 + 4 = 46$ . Points for  $D : 9 \times 1 + 6 \times 2 + 4 \times 4 = 9 + 12 + 16 = 37$ .

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• Which candidate wins?

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- Which candidate wins?
- Which candidate comes in last?

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- Which candidate wins?
- Which candidate comes in last?
- Would the outcome be different if the points were 3, 2, 1, 0?

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- Which candidate wins?
- Which candidate comes in last?
- Would the outcome be different if the points were 3, 2, 1, 0?
- What about 20, 15, 10, 5?

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- Which candidate wins?
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- Would the outcome be different if the points were 3, 2, 1, 0?
- What about 20, 15, 10, 5?
- What about 5, 4, 3, 0?

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### 6 Assignment

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- The Borda-count method is susceptible to chicanery.
- If the voters vote "honestly," then there is no problem.
- But what if...?

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### Example (Burying a Candidate)

• There are three candidates: A, the Republican; B, the Democrat; and C, the unrepentant convicted child molester who belongs to no party.

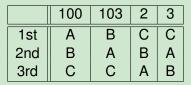
- There are three candidates: A, the Republican; B, the Democrat; and C, the unrepentant convicted child molester who belongs to no party.
- There are 208 voters.

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- There are 208 voters.
- 100 voters are Republican so they rank A first, C last.

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- There are 208 voters.
- 100 voters are Republican so they rank A first, C last.
- 103 voters are Democrats, so they rank B first, C last.

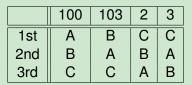
- There are three candidates: A, the Republican; B, the Democrat; and C, the unrepentant convicted child molester who belongs to no party.
- There are 208 voters.
- 100 voters are Republican so they rank A first, C last.
- 103 voters are Democrats, so they rank B first, C last.
- C has a nice-sounding name, so 5 voters rank him first.

- There are three candidates: A, the Republican; B, the Democrat; and C, the unrepentant convicted child molester who belongs to no party.
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- Their preferences:



### Example (Burying a Candidate)

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- There are 208 voters.
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- C has a nice-sounding name, so 5 voters rank him first.
- Their preferences:



Who wins?

### Example (Burying a Candidate)

- There are three candidates: A, the Republican; B, the Democrat; and C, the unrepentant convicted child molester who belongs to no party.
- There are 208 voters.
- 100 voters are Republican so they rank A first, C last.
- 103 voters are Democrats, so they rank B first, C last.
- C has a nice-sounding name, so 5 voters rank him first.

#### • Their preferences:

	100	103	2	3
1st	A	В	С	С
2nd	В	A	В	А
3rd	С	С	Α	В

• Who wins? B the Democrat wins.

- What if the Republicans decide to "bury" the Democrat?
- Their preferences:

	100	103	2	3
1st	A	В	С	С
2nd	В	Α	В	Α
3rd	С	С	А	В

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- What if the Republicans decide to "bury" the Democrat?
- Their false preferences:

	100	103	2	3
1st	Α	В	С	С
2nd	С	Α	В	A
3rd	В	С	А	В

Now who wins?

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- What if the Republicans decide to "bury" the Democrat?
- Their false preferences:

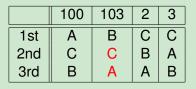
	100	103	2	3
1st	A	В	С	С
2nd	С	Α	В	A
3rd	В	С	А	В

• Now who wins? A the Republican wins because B is "buried."

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- But, what if, in addition, the Democrats also decide to "bury" the Republican?
- Their preferences:

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### Example (Burying a Candidate)

- What if, in addition, the Democrats decide to "bury" the Republican?
- Their false preferences:

	100	103	2	3
1st	A	В	С	С
2nd	С	С	В	A
3rd	В	A	Α	В

Now who wins? (A and B are both "buried.")

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### Example (Burying a Candidate)

- What if, in addition, the Democrats decide to "bury" the Republican?
- Their false preferences:

	100	103	2	3
1st	A	В	С	С
2nd	С	С	В	A
3rd	В	Α	Α	В

• Now who wins? (A and B are both "buried.") The unrepentant convicted child molester wins! Oops!

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#### Assignment

#### • Chapter 1: Exercises 11, 13, 15, 16, 21, 25, 27, 29.

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