The Plurality and Borda Count Methods

Lecture 9
Sections 1.1 - 1.3

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Outline

Definitions

Definition (The Candidates)

The candidates are the people running for office in an election. If we are choosing something other than people, we call them alternatives.

Definition (The Voters)

The voters are the people who have a say in the outcome of the election. All votes count equally.

Definition (Single-choice Ballot)

In a single-choice ballot, each voter selects one candidate.

Definition (Preference Ballot)

In a preference ballot, each voter ranks all the candidates from most preferred to least preferred.

Definition (Truncated Preference Ballot)

In a truncated preference ballot, each voter ranks some, but not all, the candidates by preference.

Definition (Single-choice Ballot)

In a single-choice ballot, each voter selects one candidate.

Definition (Preference Ballot)

In a preference ballot, each voter ranks all the candidates from most preferred to least preferred.

Definition (Truncated Preference Ballot)

In a truncated preference ballot, each voter ranks some, but not all, the candidates by preference.

- We will use preference ballots (also called ranked choice ballots).

Outline

The Math Club Election

Example

- There are four candidates for Math Club president: A, B, C, and D.
- There are 19 voting members. Their preferences are shown on the next slide.

Voters' Preferences

Example

	9	6	4
1st	A	C	D
2nd	B	B	B
3rd	C	D	A
4th	D	A	C

The preferences.

Who won?

Example

- Who should be elected president?
- Who is more popular, A or B?
- Who is more popular, A or C?
- Who is more popular, A or D?
- Who is least popular?

Who won?

Example

- Who should be elected president?
- Who is more popular, A or B?
- Who is more popular, A or C?
- Who is more popular, A or D?
- Who is least popular?
- Do “least popular” and “most unpopular” mean the same thing?

Outline

The Plurality Method

Definition (The Plurality Method)

By the plurality method, the candidate with the most *first-place* votes wins.

Example

In the Debate Club example, A wins by the plurality method.

Web Page

Run the program Voting Methods on the web.

Outline

The Borda Count Method

Definition (The Borda Count Method)

By the Borda count method, the voters rank the candidates. Then each rank is assigned points, higher ranks receiving more points. The candidate with the *most points* wins.

The Math Club Election

Example (The Math Club Election)

- Reconsider the Math Club election with 4 points for 1st, 3 for 2nd, 2 for 3rd, and 1 for 4th.

	9	6	4
1st	A	C	D
2nd	B	B	B
3rd	C	D	A
4th	D	A	C

The Math Club Election

Example (The Math Club Election)

	9	6	4
1st	A	C	D
2nd	B	B	B
3rd	C	D	A
4th	D	A	C

The Math Club Election

Example (The Math Club Election)

	9	6	4
1st	A	C	D
2nd	B	B	B
3rd	C	D	A
4th	D	A	C

Points for A : $9 \times 4 + 6 \times 1 + 4 \times 2 = 36 + 6 + 8 = 50$.

The Math Club Election

Example (The Math Club Election)

	9	6	4
1st	A	C	D
2nd	B	B	B
3rd	C	D	A
4th	D	A	C

Points for A : $9 \times 4 + 6 \times 1 + 4 \times 2 = 36 + 6 + 8 = 50$.

Points for B : $9 \times 3 + 6 \times 3 + 4 \times 3 = 27 + 18 + 12 = 57$.

The Math Club Election

Example (The Math Club Election)

	9	6	4
1st	A	C	D
2nd	B	B	B
3rd	C	D	A
4th	D	A	C

Points for A : $9 \times 4 + 6 \times 1 + 4 \times 2 = 36 + 6 + 8 = 50$.

Points for B : $9 \times 3 + 6 \times 3 + 4 \times 3 = 27 + 18 + 12 = 57$.

Points for C : $9 \times 2 + 6 \times 4 + 4 \times 1 = 18 + 24 + 4 = 46$.

The Math Club Election

Example (The Math Club Election)

	9	6	4
1st	A	C	D
2nd	B	B	B
3rd	C	D	A
4th	D	A	C

Points for A : $9 \times 4 + 6 \times 1 + 4 \times 2 = 36 + 6 + 8 = 50$.

Points for B : $9 \times 3 + 6 \times 3 + 4 \times 3 = 27 + 18 + 12 = 57$.

Points for C : $9 \times 2 + 6 \times 4 + 4 \times 1 = 18 + 24 + 4 = 46$.

Points for D : $9 \times 1 + 6 \times 2 + 4 \times 4 = 9 + 12 + 16 = 37$.

The Math Club Election

Example (The Math Club Election)

- Which candidate wins?

The Math Club Election

Example (The Math Club Election)

- Which candidate wins?
- Which candidate comes in last?

The Math Club Election

Example (The Math Club Election)

- Which candidate wins?
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- Would the outcome be different if the points were 3, 2, 1, 0?

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- What about 20, 15, 10, 5?

The Math Club Election

Example (The Math Club Election)

- Which candidate wins?
- Which candidate comes in last?
- Would the outcome be different if the points were 3, 2, 1, 0?
- What about 20, 15, 10, 5?
- What about 5, 4, 3, 0?

Outline

Burying a Candidate

Burying a Candidate

- The Borda-count method is susceptible to chicanery.
- If the voters vote “honestly,” then there is no problem.
- But what if. . . ?

Burying a Candidate

Example (Burying a Candidate)

- There are three candidates: A, the Republican; B, the Democrat; and C, the unrepentant convicted child molester who belongs to no party.

Burying a Candidate

Example (Burying a Candidate)

- There are three candidates: A, the Republican; B, the Democrat; and C, the unrepentant convicted child molester who belongs to no party.
- There are 208 voters.

Burying a Candidate

Example (Burying a Candidate)

- There are three candidates: A, the Republican; B, the Democrat; and C, the unrepentant convicted child molester who belongs to no party.
- There are 208 voters.
- 100 voters are Republican so they rank A first, C last.

Burying a Candidate

Example (Burying a Candidate)

- There are three candidates: A, the Republican; B, the Democrat; and C, the unrepentant convicted child molester who belongs to no party.
- There are 208 voters.
- 100 voters are Republican so they rank A first, C last.
- 103 voters are Democrats, so they rank B first, C last.

Burying a Candidate

Example (Burying a Candidate)

- There are three candidates: A, the Republican; B, the Democrat; and C, the unrepentant convicted child molester who belongs to no party.
- There are 208 voters.
- 100 voters are Republican so they rank A first, C last.
- 103 voters are Democrats, so they rank B first, C last.
- C has a nice-sounding name, so 5 voters rank him first.

Burying a Candidate

Example (Burying a Candidate)

- There are three candidates: A, the Republican; B, the Democrat; and C, the unrepentant convicted child molester who belongs to no party.
- There are 208 voters.
- 100 voters are Republican so they rank A first, C last.
- 103 voters are Democrats, so they rank B first, C last.
- C has a nice-sounding name, so 5 voters rank him first.
- Their preferences:

	100	103	2	3
1st	A	B	C	C
2nd	B	A	B	A
3rd	C	C	A	B

Burying a Candidate

Example (Burying a Candidate)

- There are three candidates: A, the Republican; B, the Democrat; and C, the unrepentant convicted child molester who belongs to no party.
- There are 208 voters.
- 100 voters are Republican so they rank A first, C last.
- 103 voters are Democrats, so they rank B first, C last.
- C has a nice-sounding name, so 5 voters rank him first.
- Their preferences:

	100	103	2	3
1st	A	B	C	C
2nd	B	A	B	A
3rd	C	C	A	B

- Who wins?

Burying a Candidate

Example (Burying a Candidate)

- There are three candidates: A, the Republican; B, the Democrat; and C, the unrepentant convicted child molester who belongs to no party.
- There are 208 voters.
- 100 voters are Republican so they rank A first, C last.
- 103 voters are Democrats, so they rank B first, C last.
- C has a nice-sounding name, so 5 voters rank him first.
- Their preferences:

	100	103	2	3
1st	A	B	C	C
2nd	B	A	B	A
3rd	C	C	A	B

- Who wins? B the Democrat wins.

Burying a Candidate

Example (Burying a Candidate)

- What if the Republicans decide to “bury” the Democrat?
- Their preferences:

	100	103	2	3
1st	A	B	C	C
2nd	B	A	B	A
3rd	C	C	A	B

- . . .

Burying a Candidate

Example (Burying a Candidate)

- What if the Republicans decide to “bury” the Democrat?
- Their false preferences:

	100	103	2	3
1st	A	B	C	C
2nd	C	A	B	A
3rd	B	C	A	B

- Now who wins?

Burying a Candidate

Example (Burying a Candidate)

- What if the Republicans decide to “bury” the Democrat?
- Their false preferences:

	100	103	2	3
1st	A	B	C	C
2nd	C	A	B	A
3rd	B	C	A	B

- Now who wins? A the Republican wins because B is “buried.”

Burying a Candidate

Example (Burying a Candidate)

- But, what if, in addition, the Democrats also decide to “bury” the Republican?
- Their preferences:

	100	103	2	3
1st	A	B	C	C
2nd	C	C	B	A
3rd	B	A	A	B

- . . .

Burying a Candidate

Example (Burying a Candidate)

- What if, in addition, the Democrats decide to "bury" the Republican?
- Their false preferences:

	100	103	2	3
1st	A	B	C	C
2nd	C	C	B	A
3rd	B	A	A	B

- Now who wins? (A and B are both "buried.")

Burying a Candidate

Example (Burying a Candidate)

- What if, in addition, the Democrats decide to "bury" the Republican?
- Their false preferences:

	100	103	2	3
1st	A	B	C	C
2nd	C	C	B	A
3rd	B	A	A	B

- Now who wins? (A and B are both "buried.") The unrepentant convicted child molester wins! Oops!

Outline

Assignment

Assignment

- Chapter 1: Exercises 11, 13, 15, 16, 21, 25, 27, 29.